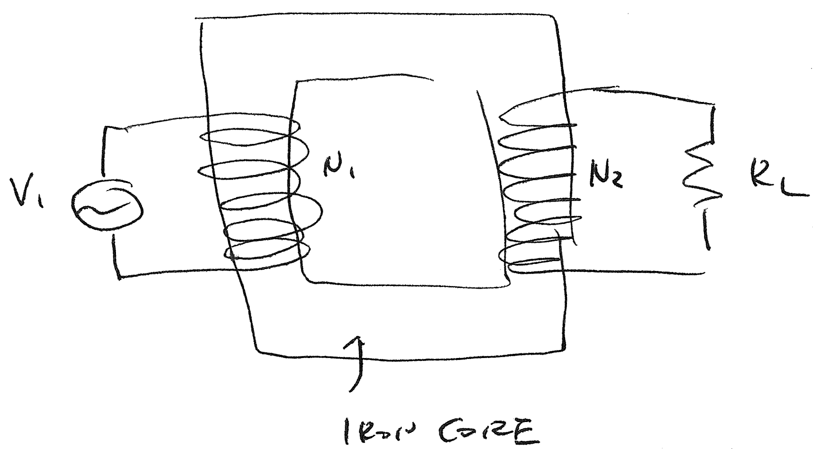
TRANSFORMERS



$$V_1 = -N_1 \frac{d\Phi_B}{dt} \Rightarrow V_2 = -N_2 \frac{d\Phi_B}{dt}$$

$$\frac{d\Phi_B}{dt} = -\frac{V_1}{N_1}$$

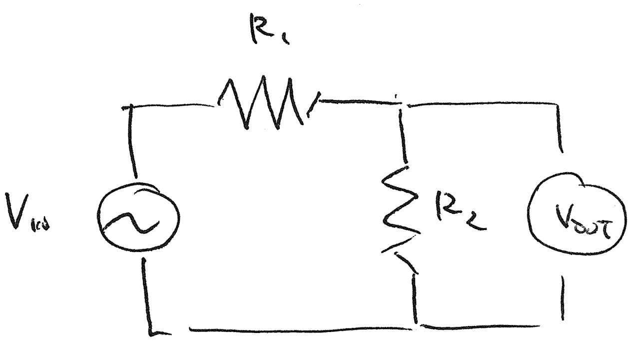
$$V_2 = -N_2 \cdot -\frac{V_1}{N_1} = \frac{N_2}{N_1} V_1$$

$N_2 > N_1$: STEP UP TRANSFORMER

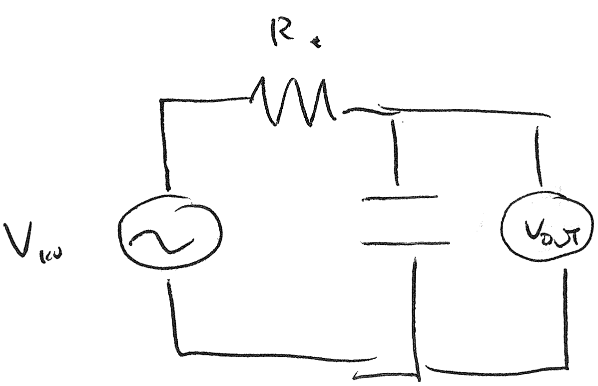
$N_1 < N_2$: STEP DOWN TRANSFORMER

FILTERS

VOLTAGE DIVIDER



$$V_{out} = V_{in} \cdot \frac{R_2}{R_1 + R_2}$$



$$V_{out} = V_{in} \frac{\frac{1}{i\omega C}}{R^2 + \frac{1}{i\omega C}}$$

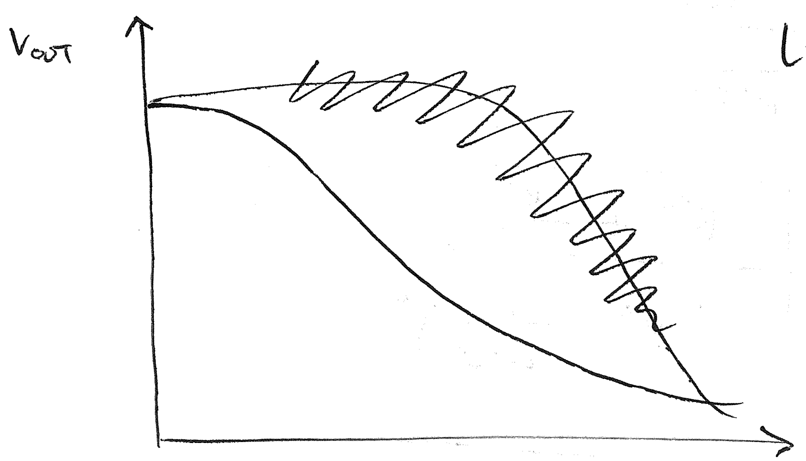
$$= V_{in} \frac{\frac{1}{i\omega C}\left(R - \frac{1}{i\omega C}\right)}{R^2 + \frac{1}{\omega^2 C^2}}$$

$$= V_{in} \frac{\frac{R}{i\omega C} + \frac{1}{\omega^2 C}}{R^2 + \frac{1}{\omega^2 C^2}}$$

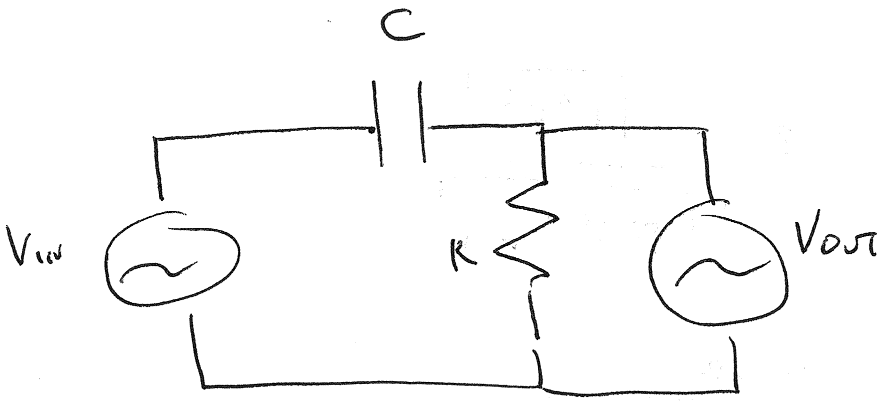
$$V_{out} = V_{in} \frac{1 - i\omega C R}{\omega^2 C^2 R^2 + 1}$$

$$|V_{OUT}| = |V_{IN}| \sqrt{\frac{1 + \omega^2 C^2 R^2}{(1 + \omega^2 C^2 R^2)^2}}$$

$$\boxed{V_{OUT} = |V_{IN}| \sqrt{\frac{1}{1 + \omega^2 C^2 R^2}}}$$



LOW PASS FILTER



$$V_{out} = V_{in} \frac{R}{R + \frac{1}{i\omega C}}$$

$$V_{out} = V_{in} \frac{R}{R^2 + \frac{1}{\omega^2 C^2}} \cancel{(1+} \left(R - \frac{1}{i\omega C} \right)$$

$$= V_{in} \frac{R^2 - \frac{R}{i\omega C}}{R^2 + \frac{1}{\omega^2 C^2}}$$

$$|V_{out}| = |V_{in}| \sqrt{\frac{R^4 + \frac{R^2}{\omega^2 C^2}}{\left(R^2 + \frac{1}{\omega^2 C^2} \right)^2}}$$

$$= |V_{in}| \; R \sqrt{\frac{1}{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$= |V_{in}| \; R \frac{\sqrt{\omega^2 C^2}}{\sqrt{R^2 \omega^2 C^2 + 1}}$$

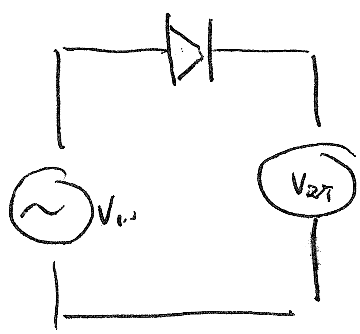
$$V_{out} = V_{in} \frac{\omega C R}{\sqrt{1 + R^2 \omega^2 C^2}}$$

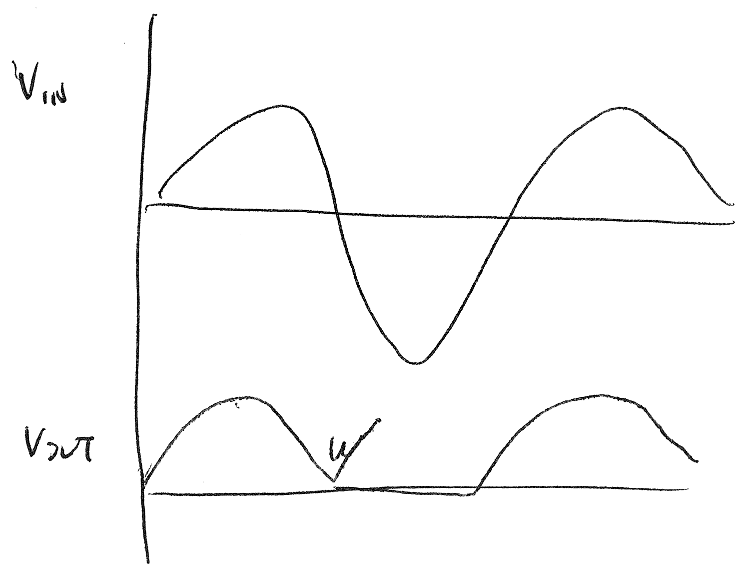
$V_{out} \to 0 \qquad \omega \to 0$

$V_{out} = V_{in} \qquad \omega \to \infty$

HIGH PASS FILTER

DIODE





POWER EXAMPLE

20 MW POWER PLANT

COMMON VOLTAGE ~~≈~~ 20 kV (RMS)

STEPPED UP TO 200 kV FOR TRANSMISSION (RMS)

RESISTANCE OF 1 km OF WIRE IS 2Ω. ENERGY COST ~ 10¢/kWhr WHAT IS ENERGY COST LOST TO WIRES?

~~$P_{AVG} = I_{RMS}^2 R = \frac{V_{RMS}^2}{R}$~~

~~$I_{RMS} =$~~

$$P_{AVG} = I_{RMS} V_{RMS} \Rightarrow 20 \times \overset{10^3}{\cancel{10^6}}\, W = (200 \times \cancel{10^3}\, V)\, I_{RMS}$$

$$I_{RMS} = 100\, A$$

$$P_{WIRES} = I_{RMS}^2 R = (100\, A)^2 (2) = 20\, kW$$

$$U_{WIRES} = (20\, kW)\ 24\, h = 480\, kWhr$$

$$480\, kWhr \times 10¢/kWhr = \$48$$

WHAT HAPPENS IF IT'S NOT STEPPED UP ?

$20\,MW = (20\,KV)\ I_{lines}$

$\frac{20 \times 10^6}{20 \times 10^3} = 1000\,A$

$P_{WIRES} = (1000\,A)^2 \cdot 2 = 2\,MW$

$U = 48\,Mwhr$

$GST = \cancel{48 \times 10^6\ kwhr}$

$= 48 \times 10^3\ kwhr \times 0.1/kwhr$

$= \$14800$

EM WAVES

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{ENCLOSED}}{\varepsilon_0}$$

$$\oint \vec{B}\cdot d\vec{A} = 0$$ NO MONOPOLES

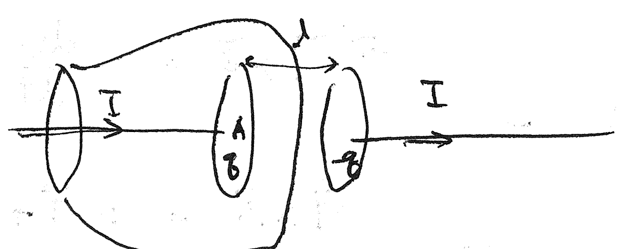
$$\oint \vec{E}\cdot ds = -\frac{d\Phi_B}{dt}$$ FARADAYS LAW

$\int$ . . . .

1.

AMPERE'S LAW

CONTRADICTION?



$$\frac{dq}{dt} = I \qquad \frac{q}{A} = \sigma$$

$$\frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0} = E$$

$$I = \frac{dq}{dt} = \frac{d}{dt}A\varepsilon_0 E = A\varepsilon_0\frac{dE}{dt}$$

AMPERE'S LAW $\oint \vec{B}ds = \mu_0 I$

$$\mu_0 I \Rightarrow \mu_0\left(I + \varepsilon_0 \frac{d\Phi_E}{dt}\right)$$

$$\boxed{\oint B \cdot ds = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{ENCLOSED}}{\varepsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint E \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \cdot B = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{dB}{dt}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

IN FREE SPACE

$\rho = 0$

$\vec{J} = 0$

$$\oint \vec{E} \cdot d\vec{s} = \cancel{\oint \vec{\nabla} \times \vec{E}\, d\vec{A}} = \int (\vec{\nabla} \times \vec{E})\, d\vec{A}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B}) = -\frac{\partial}{\partial t}\Big(\underbrace{\cancel{\mu_0 \vec{J}}}_{0} + \mu_0\varepsilon_0 \frac{\partial E}{\partial t}\Big)$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 E$$

$$-\nabla^2 E = -\mu_0 \varepsilon_0 \frac{d^2 \vec{E}}{dt^2}$$

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{d^2 E}{dt^2}$$

In 1D

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$E(x) = E_0 \sin(kx - \omega t)$$

$$k^2 = \mu_0 \varepsilon_0 \omega^2$$

$$\frac{\omega^2}{k^2} = \frac{1}{\mu_0 \varepsilon_0} \qquad \frac{1}{\mu_0 \varepsilon_0} = c^2$$

$$\boxed{\lambda f = c}$$

HIGHER FREQUENCY = LOWER WAVELENGTH

WIFI 2.4GHz

$c = 3.0 \times 10^8 \, m/s$

$$\lambda = \frac{3.0 \times 10^8 \, m/s}{2.4 \times 10^9} = \frac{\cancel{30}^{10}}{\cancel{24}\,8} \Rightarrow 1.25 \times 10^{-1} \, m$$

$= 0.125 \, m$

RADIO WAVES $\lambda = 10^4 \rightarrow 0.1 \, m$

MICROWAVE $\lambda = 0.3$ TO $10^{-4} \, m$

INFRARED $\lambda = 10^{-3}$ TO $7 \times 10^{-7} \, m$

1mm TO 700nm

VISIBLE $\lambda =$ 400 nm TO 700nm

VIOLET RED

ULTRAVIOLET $\lambda =$ 6 Å TO 400nm

X RAY $\lambda = 10^{-8}$ TO $10^{-12} \, m$

γ RAY $\lambda = 10^{-10}$ TO $10^{-14} \, m$

$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$

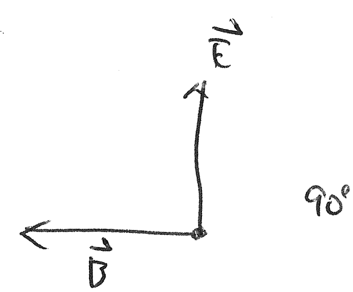
★ LIGHT CARRIES ENERGY

⇓

SOLAR ENERGY

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t}$$

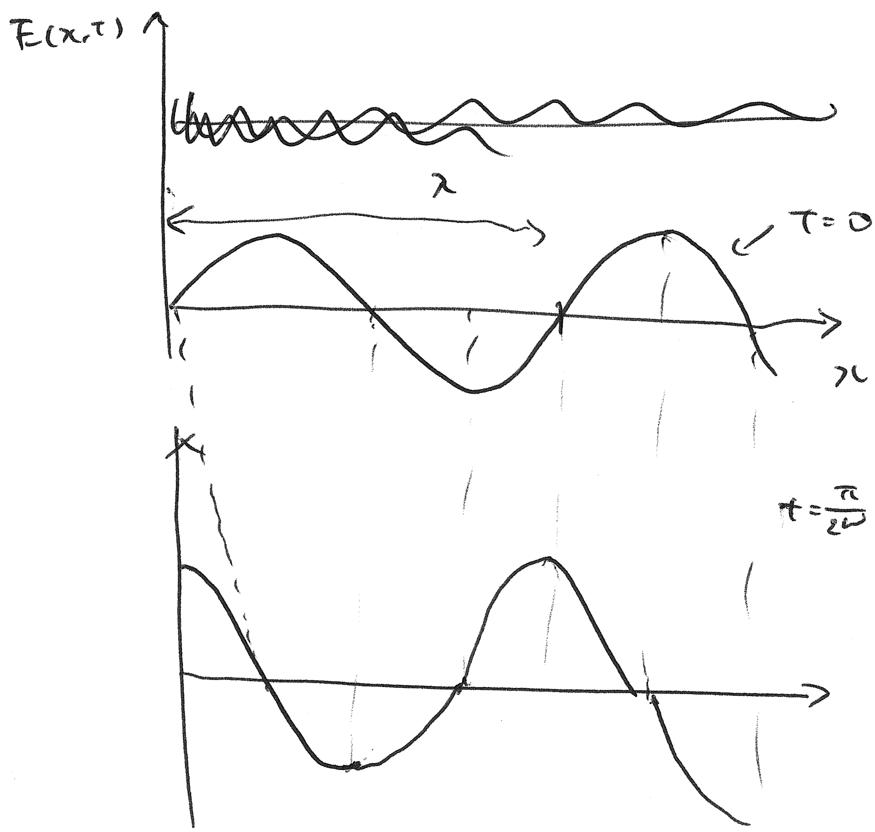


$\vec{E} \times \vec{B}$: POINTS IN THE DIRECTION OF WAVE TRAVEL

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

TRIAL SOLUTION

$$E(x,t) = E_0 \sin(kx - \omega t)$$



$\lambda = \frac{2\pi}{k}$

$\rightarrow \quad k = \frac{2\pi}{\lambda}$

ω

$$\omega = 2\pi f$$

$$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \underline{\underline{\lambda f}}$$

$$\lambda f = v$$

$$\frac{\omega}{k} = v$$

SPEED OF THE LIGHT

~~SPEED~~ LHT

TRAVELING WAVE SOLUTION

$$\frac{\partial^2 E}{\partial x^2} = -E_0 k^2 \sin(kx - \omega t)$$

$$\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = -\mu_0 \varepsilon_0 E_0 \omega^2 \sin(kx - \omega t)$$

$$\therefore \quad \cancel{E_0} k^2 = \mu_0 \varepsilon_0 \cancel{E_0} \omega^2$$

$$\frac{\omega^2}{k^2} = \frac{1}{\mu_0 \varepsilon_0}$$

$$E(x,T) = E_{MAX} \sin(kx - \omega T)$$

$$B(x,T) = B_{MAX} \sin(kx - \omega T)$$

$$\vec{\nabla} \times \vec{E} = -\frac{dB}{dT}$$

$$\frac{dE}{\delta x} = -\frac{dB}{dT}$$

$$\frac{dE}{dx} = E_{MAX}\, k \cos(kx - \omega T)$$

$$-\frac{dB}{dT} = +B_{MAX}\, \omega \cos(kx - \omega T)$$

$$E_{MAX}\, k = B_{MAX}\, \omega$$

$$\frac{E_{MAX}}{B_{MAX}} = \frac{\omega}{k} = c$$